Quiz 6, Linear

Name: ____________________________________

1. (4 points) Let W be the set of all vectors of the form $\begin{bmatrix} 2a-3b \\ -1 \\ 2a-5b \end{bmatrix}$, where a, b are arbitrary real numbers. Either find a set S that spans W or give an example (or explanation) to show that W is *not* a vector space.

2. (3 points) Suppose $\mathbb{R}^4 = \text{Span}\,\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$. *Use the definition of basis*, as well as the Invertible Matrix Theorem, to explain why $\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$ is a basis for $\mathbb{R}^4$.

3. (3 points) Let $M_{2\times 2}$ be the vector space of all 2×2 matrices, and define $T : M_{2\times 2} \to M_{2\times 2}$ by $T(A) = A + A^T$, where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Show that T is a linear transformation.

4. (1 point, Extra Credit) Let $A = \begin{bmatrix} 2 & 3 & -2 & 0 & 4 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$. Find a nonzero vector in Nul A and a nonzero vector in Col A. Explain why your answers are correct.